MATHEMATICAL MODELS FOR FORECASTING THE STATE OF THE ENVIRONMENT

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The system to provide the safety of potentially dangerous objects of the city economy created by the united information city system includes two subsystems: monitoring subsystem of the environment state and the subsystem of forecasting the main purpose of which is the warning of extraordinary situations [1]. Some experience shows that more often arising extraordinary situations are connected with the wastes of polluting substances thrown to the environment: to the atmosphere, water and soil.

The warning of the extraordinary situations is based on the information of the current environment state and on forecasting this state in perspective. From the point of designing and elaborating the forecasting subsystem to use these models directly i.e. as they are formulated is not reasonable owing to the following reasons.

All authors consider only one of the indicated environments. Because of this reason they differ both methodic of model building and necessary data for it. By designing and using the forecasting system it would be desirable to have one generalized model so as separate models would enter as a private event. It would be possible to get them from the generalized model by giving them some parameters. Such possibility exists since the mathematical models describing the concentration dynamics of polluting substances in the air, water and soil are based on the laws of mass conservation and characterized by the differential equations in quotient derivatives [2].

One of such equations is the equation of substance diffusion in the homogeneous (ambience) environment (the equation of Fick):

$$\frac{\partial c}{\partial t} = \frac{d}{dx}\left(k_x \frac{\partial c}{\partial x}\right) + \frac{d}{dy}\left(k_y \frac{\partial c}{\partial y}\right) + \frac{d}{dz}\left(k_z \frac{\partial c}{\partial z}\right) - -V_x \frac{\partial c}{\partial x} - V_y \frac{\partial c}{\partial y} - V_z \frac{\partial c}{\partial z} + q(t), \quad (1)$$

where k_x, k_y, k_z – are diffusion coefficients;
V_x, V_y, V_z – is the velocity of flow movement in the corresponding directions (for the when the field is spread in this flow); case
$q(t)$ – is the intensity of polluting substance wastes (the function of the polluting source);
c – is the concentration of polluting substance admixtures;
x, y, z – are rectangular coordinates.

Owing to the Stated reasons the application of the dynamic models with distributed parameters is reasonable. It enables to proceed from the unceasing task to its discrete type and completely use it for receiving the functional dependency describing the process of spreading polluting substances in the environment, the information accumulated in the frames of the monitoring system.

It can be defined as the synthesis of a widespread statistical and analytical approach, therefore it is possible to expect it to be more perspective.

The statistical information is used for building the models the structure of which corresponds to physical notions of modulated process. In the process of modulating the model structure can be corrected for the best display of statistical data. It provides the sufficient freedom of the choice.

One of the variants to build the mathematical model of the field to pollute the city atmosphere by using the adaptation principle is the method of group argument account. The example of using MGAA for solving the tasks of modeling to pollute the air basin is considered in [3]. However the possibility of forecasting the change of polluting substance concentration was not considered here at time.

In the given work the main attention is paid just to the questions of forecasting the changes of the environment state on the definite time interval with account that the field of polluting substance concentration is not stationary and heterogeneous and depends both on external factors and on the prehistory of the process development.

Being designed for these purposes the model building methodic includes some steps. First an algebraically interfiled model is built. Herewith there are no restrictions of the quantitative and qualitative compositions of predicates taken into account by a user in the model that is he himself can independently decide what meteoparameters or predicates of another type will be used in the model. Moreover a user is given the possibility to form new predicates on the base of available. A user can also choose a model type on the first stage. This model type may be: linear, sedate or significant.

Let the meaning of the polluting substance concentration q is defined as a certain function $f(\overline{x}, \overline{p})$ depending on $\overline{x}$ –vector of spatial coordinates and $\overline{p}$ – vector of meteoparameters then the model of the lineal type is defined as:

$$q(\overline{x}, \overline{p}) = a_0 + \sum_{i=1}^{3} a_i \cdot x_i + \sum_{j=1}^{m} a_j \cdot p_j , \quad (2)$$

where m – is a quantity of accounted meteoparameters;
a_i and a_j – are model coefficients;
a_0 – is a free term.

Sedate model:

$$q(\bar{x},\bar{p}) = a_0 \cdot \prod_{i=1}^{3} x_i^{a_i} \cdot \prod_{j=1}^{m} p_j^{a_j}, \qquad (3)$$

significant model:

$$q(\bar{x},\bar{p}) = a_0 \cdot \prod_{i=1}^{3} a_i^{x_i} \cdot \prod_{j=1}^{m} a_j^{p_j}, \qquad (4)$$

Thus, the algebraic interpolation model is built. It serves the base of building finally difference model on the second stage.

Herewith the forecasting region is covered with an even rectangular net the knot number of which or a step according to corresponding coordinated axis's are defined by a user.

While building so called "exact" finally difference model a pattern is used. It is defined as follows:

$$q(t_{+1},x_0,y_0) = c_0 + c_1 \cdot q(t_0,x_0,y_0) + c_2 \times$$
$$\times q(t_{-1},x_0,y_0) + c_3 \cdot q(t_{-2},x_0,y_0) +$$
$$+ c_4 \cdot q(t_0,x_{-1},y_0) + c_5 \cdot q(t_0,x_{+1},y_0) + c_6 \times$$
$$\times q(t_0,x_0,y_{-1}) + c_7 \cdot q(t_0,x_0,y_{+1}),$$

where $q(t_i, x_j, y_k)$ – are the meanings of admixture concentration in the point with coordinates x_j, y_k, defined as follows:

$$x_j = x_0 + j \cdot \Delta x, \quad y_k = y_0 + k \cdot \Delta y, \quad t_i = t_0 + i \cdot \Delta t$$

$(j,k = -1, 0, +1)$, $(I = -2, -1, 0, +1)$.

The next stage is processing the statistical information. As external selection criteria the following can be used by a user's choice:

– the criterion of regularity at the examination

$$CS = \frac{\sum_{i=1}^{N_A} (q_{w_i} - \hat{q}_{w/a_i})^2 + (q_{w_i} - \hat{q}_{w/b_i})^2}{\sum_{i=1}^{N_\cup} q_{w_i}^2}, \qquad (5)$$

– the criterion of undisplacement

$$RS = \frac{\sum_{i=1}^{N_c} (q_{c_i} - \hat{q}_{c/w_i})^2}{\sum_{i=1}^{N_c} q_{c_i}^2}, \qquad (6)$$

– the criterion of forecast stability

$$SNS = \frac{\sum_{i=1}^{N_B} (q_{w/b_i} - \hat{q}_{w/b_i})^2}{\sum_{i=1}^{N_b} q_{w_i}^2}, \qquad (7)$$

– two combined criteria

$$i = \frac{\sum_{i=1}^{N_B} (q_{b_i} - \hat{q}_{b/w_i})^2}{\sum_{i=1}^{N_b} q_{b_i}^2}. \qquad (8)$$

1. Undisplacement + regularity:

$$\sqrt{SNS + RS} \qquad (9)$$

2. Undisplacement + stability:

$$\sqrt{SNS + i} \qquad (10)$$

The following signs are used here:

N_C – is a point number in subset of examination point's C;

N_B – is a point number in subset of checking point's B;

N_A – is a point number in subset of training point's A; herewith the equation $N = N_A + N_B + N_C$ is performed N is a point number of the starting multitude observations F, $F = A \cup B \cup C$; the unification of subsets $A \cup B$ is an extraction (W) with the volume $NW = NA + NB$;

q_{wi} – is the meaning of the polluting substance concentration got as a result of observations, $q_{wi} \subset W$, $I = 1..N_W$; q_{ai} – is the meaning of the polluting substance concentration got as a result of observations, $q_{ai} \subset A$, $I = 1..N_A$;

q_{bi} – is the meaning of the polluting substance concentration got as a result of observations, $q_{bi} \subset B$, $I = 1..N_B$;

q_{ci} – is the meaning of the polluting substance concentration got as a result of observations;

$q_{ai} \subset C$, $I = 1..N_C$; $\hat{q}_{w/a_i}$ – is the meaning of polluting substance concentration calculated for the points of extraction W by the models whose coefficients were defined the points of extraction A;

$\hat{q}_{w/b_i}$ – is the meaning of polluting substance concentration calculated for the points of extraction W by the models whose coefficients were defined the points of extraction B;

$\hat{q}_{c/w_i}$ – is the meaning of polluting substance concentration calculated for the points of extraction C by the models whose coefficients were defined the points of extraction W.

The possibility to use several types of algebraic model for interpolation (lineal, sedate, significant) designing enables the person modeling the process of spreading polluting substance admixture to lay his own ideas about this process and account the meteoparameters which are at his disposal.

As a result of the program work the algebraic interpolation model of the following type was built:

$$q(x',y',v) = 2{,}8337 \cdot 1{,}0135^{-(x')^2} \times$$
$$\times (7-y')^{2{,}037} \cdot -0{,}3^{v} \cdot e^{0{,}997 \cdot (7-y')}. \qquad (11)$$

On its base two optimum finally difference models by symmetrical criterion of stability were received:

1) the criterion meaning: 0,006477

the meaning CKO: 0,0512

$$q(t_{+1},x_0,y_0) = -0,2322 + 21,54 \cdot q(t_0,x_0,y_0) + 0,425 \cdot q(t_{-1},x_0,y_0) - 0,172 \cdot q(t_{-2},x_0,y_0) - -1,044 \cdot q(t_0,x_{-1},y_0) - 0,657 \cdot q(t_0,x_{+1},y_0) - 9,66 \cdot q(t_0,x_0,y_{-1}) - 9,457 \cdot q(t_0,x_0,y_{+1})$$

2) the criterion meaning: 0,006708
the meaning CKO: 0,0526

$$q(t_{+1},x_0,y_0) = -0,631 + 0,87 \cdot q(t_0,x_0,y_0) + 0,434 \cdot q(t_{-1},x_0,y_0) - 0,139 \cdot q(t_{-2},x_0,y_0) - -0,227 \cdot q(t_0,x_{-1},y_0) + 0,106 \cdot q(t_0,x_{+1},y_0) - 0,007 \cdot q(t_0,x_0,y_{-1}).$$

In conclusion we should not that the proposed methodic and realized program module on its base enables to build the forecasting field model of atmosphere pollution. Using algorithms by the method of group argument account permits flexible optimum choice of the model structure by the user's indicated criterion and its predictors. The program realizing these two methodic may be easily adapted to build the models of the similar class and for other environments (for instance: water and soil). It is a premise for creating the united unified system of building forecasting models of polluting the environment.

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